

We can use sinusoidal functions to model periodic data that does not involve angles.

Ex. The following table lists the number of hours of daylight on the first day of each month in Toronto.

- Determine an equation to model the data.
- Graph the model and data on the same set of axes. Comment on the fit.
- Use the model to predict the number of hours of daylight on September 15<sup>th</sup>.

Month	1	2	3	4	5	6	7	8	9	10	11	12	13
Hours	9:00	9:54	11:11	12:44	14:09	15:13	15:23	14:34	13:12	11:44	10:15	9:11	9:00
Hours	9.0	9.9	11.2	12.7	14.15	15.2	15.4	14.6	13.2	11.7	10.8	9.2	9.0

$$\text{Vert. shift: } \frac{\text{max} + \text{min}}{2} = \frac{15.4 + 9}{2} = 12.2$$

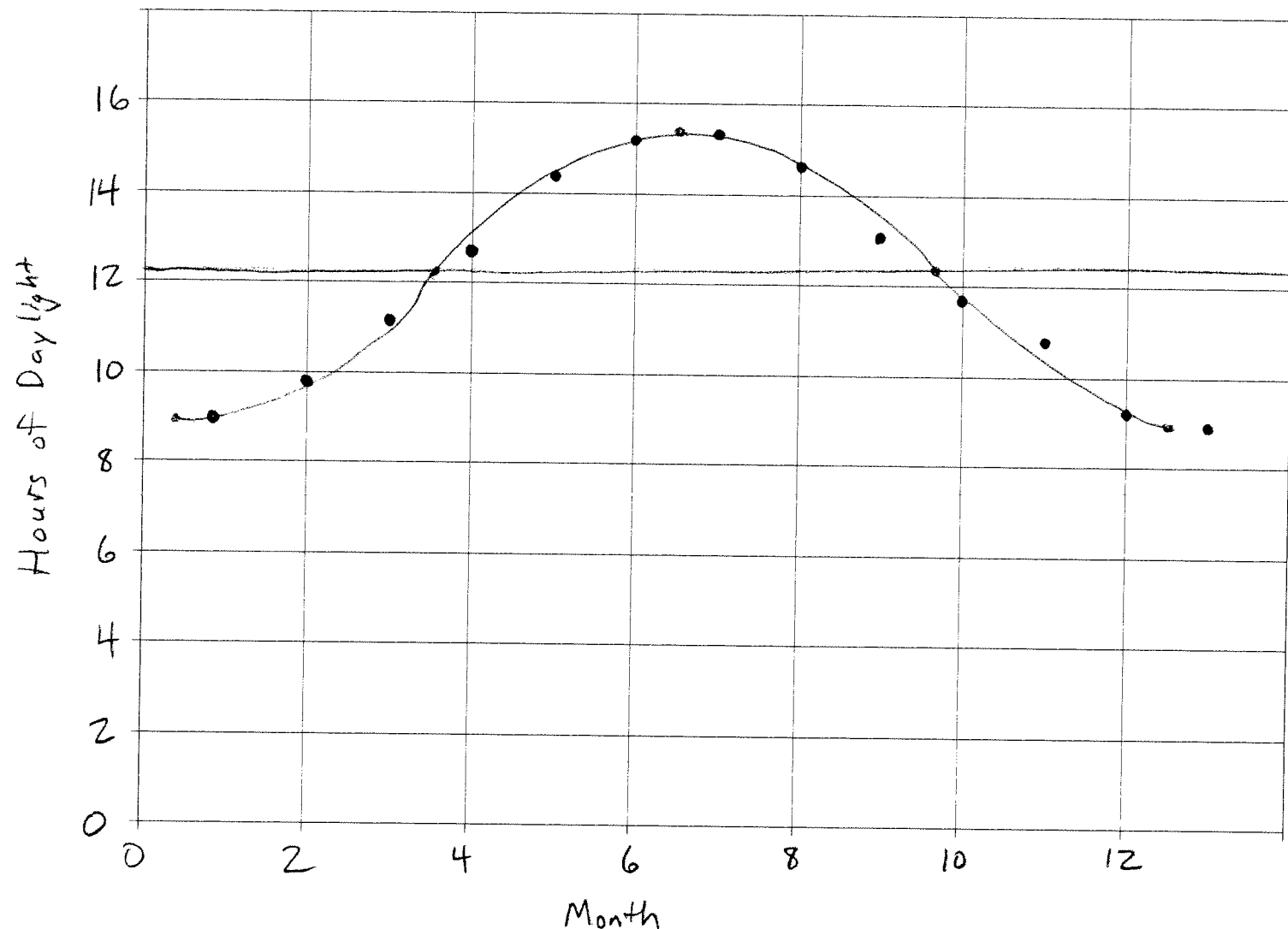
$$\text{Amplitude: } \frac{\text{max} - \text{min}}{2} = \frac{15.4 - 9}{2} = 3.2$$

$$\text{Period: } 12 \quad (\text{it's a yearly pattern!})$$

$$\text{Phase shift: } \begin{array}{l} \sin: 3.5 \text{ right} \\ \cos: 7 \text{ right} \end{array}$$

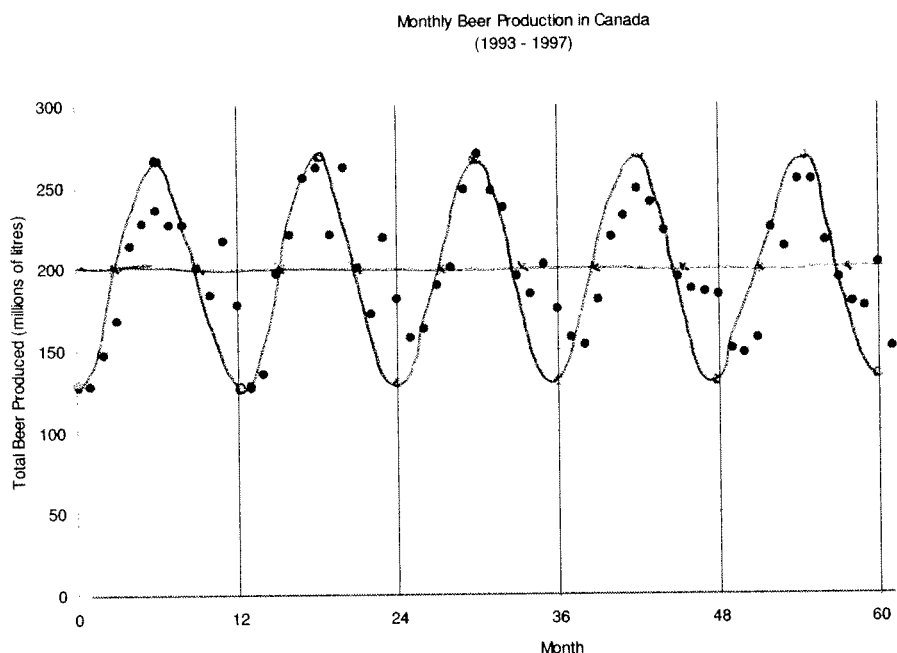
$$y = 3.2 \sin[30(x - 3.5)] + 12.2$$

$$\text{or } y = 3.2 \cos[30(x - 7)] + 12.2$$



Ex. Monthly beer production in Canada is shown in the table and graph below, beginning in January 1993.

Month	Millions of L
1	128
2	147
3	168
4	214
5	228
6	236
7	227
8	227
9	200
10	184
11	217
12	177
13	127
14	135
15	197
16	220
17	255
18	262
19	220
20	262
21	200
22	172
23	219
24	181
25	157
26	163
27	189
28	200
29	248
30	270
31	247
32	237
33	195
34	184
35	202
36	175
37	157
38	153
39	180
40	219
41	231
42	248
43	240
44	222
45	194
46	187
47	185
48	183
49	150
50	147
51	156
52	224
53	212
54	253
55	253
56	216
57	193
58	178
59	176
60	202
61	151



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- Determine an equation to model this data.
- Graph the model on the same set of axes as the data.
- Describe any discrepancies between the model and the data.

Vert. Shift:  $\frac{\text{max} + \text{min}}{2} = \frac{270 + 128}{2} = 199$

Amplitude:  $\frac{\text{max} - \text{min}}{2} = \frac{270 - 128}{2} = 71$

Period: 12 (yearly pattern!)

Phase shift: sin: 3.5 right  
cos: 6 right