MCR3U

3.3 Rational exponents

The n^{th} root is the opposite operation of the exponent n: $x^{\frac{1}{n}} = \sqrt[n]{x}$

- the exponent "2" represents "squared" - the exponent "1/2" represents "square root"
- the exponent "3" represents "cubed" - the exponent "1/3" represents "cube root"

Ex. a) Calculate the volume of a cube that has a side length 5 cm.

b) Calculate the side length of a cube with volume 64 cm^3.

Ex. Evaluate each root. Write your answers as integers or fractions.

a) $81^{\frac{1}{2}}$ b) $125^{\frac{1}{3}}$ c) $\sqrt[5]{-32}$ d) $\sqrt[3]{\frac{27}{1000}}$

Ex. Explain how $8^{\frac{1}{3}}$ is different from 8^{-3}.

The rational exponent involves a numerator and a denominator: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$

- the numerator represents the "normal" exponent - "to the exponent m"
- the denominator represents the "root" exponent - "the n^{th} root"

Ex. Evaluate each expression. Show at least one step of algebraic reasoning.

a) $8^{\frac{5}{3}}$ b) $125^{\frac{2}{3}}$ c) $27^{-\frac{1}{3}}$ d) $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$

p. 175 # 1, 2, 3, 4, 7.

Radioactive decay can be modelled by an exponential function for the mass of a substance as it decays.

$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ where

$A(t)$ is the amount or mass remaining after time t

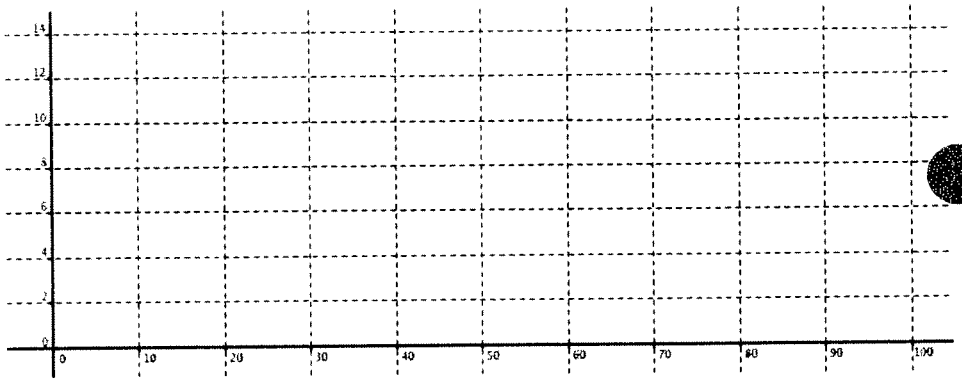
A_0 is the original amount or mass when $t = 0$

h is the half-life of the element, measured in the same units as t

Half-life h of a radioactive element is the time required for a sample to decay to half its original amount.

* t and h must be measured in the same units (seconds, minutes, hours, or years)

Ex. The element platinum-197 has a half-life of 20 hours. Complete a table of values and sketch a graph to show the amount of platinum-197 remaining in a 12 mg sample until there is less than 1 mg remaining.



Ex. Radon-219 has a half-life of 4 seconds. What mass of a 50 g sample remains after 30 seconds?

Homework:

1. Iodine-131 has a half-life of 8 days. What mass of a 50 g sample remains after 30 days?

2. Cerium-143 has a half-life of 33 hours. What mass of a 100 g sample remains after one week?

3. Calculate the volume of a sphere that has a surface area of 100 m^2, given the relation $V = \frac{S^{\frac{3}{2}}}{6\sqrt{\pi}}$.

4. Determine the value of x in the expression $\left(\sqrt[3]{x}\right) = 4$. Show your work.

5. Determine the value of x in the expression $x^{\frac{4}{3}} = 256$. Show your work.