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Name: Answers Date: _____

MCR3U

Diagnostic quiz: Exponential functions

1. Evaluate each expression. Show your work and write your answers as fractions in lowest terms.

$$\begin{aligned} \text{a) } \left(\frac{1}{27}\right)^{\frac{4}{3}} &= \left(\sqrt[3]{\frac{1}{27}}\right)^4 \\ &= \left(\frac{\sqrt[3]{1}}{\sqrt[3]{27}}\right)^4 \\ &= \left(\frac{1}{3}\right)^4 \\ &= \frac{1}{81} \end{aligned}$$

$$\begin{aligned} \text{b) } 3^{-2} + 5^{-1} + 7^0 \\ &= \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^1 + 1 \\ &= \frac{1}{9} + \frac{1}{5} + 1 \\ &= \frac{5}{45} + \frac{9}{45} + \frac{45}{45} \\ &= \frac{59}{45} \end{aligned}$$

2. Simplify each expression. Write your answers using only positive exponents.

$$\begin{aligned} \text{a) } (2x)^4(3x)^{-1} \\ &= 2^4 x^4 \cdot \frac{1}{3x} \\ &= \frac{16x^4}{3x} \\ &= \frac{16x^3}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{4(3x)^2}{2x^5} &= \frac{4 \cdot 3^2 x^2}{2x^5} \\ &= \frac{4 \cdot 9x^2}{2x^5} \\ &= \frac{36x^2}{2x^5} \\ &= \frac{18}{x^3} \end{aligned}$$

3. Explain how $8^{\frac{1}{3}}$ is different from 8^{-3} .

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$8^{-3} = \left(\frac{1}{8}\right)^3 = \frac{1}{512}$$

4. Explain why ONE of the following statements is incorrect:

a) $4^3 + 4^{-1} = 4^2$

Can't add exponents when terms are added!

b) $5^x 5^x = 25^{2x}$

Don't multiply the bases together!

c) $8(4^x) = 32^x$

Wrong on many levels.

→ Should change $8 \cdot 4$ to a common base!

5. Write an equation for the exponential function that results from each set of transformations applied to the base graph $y = 10^x$.

a) shifted right 4 units and reflected horizontally

b) compressed vertically by $\frac{1}{4}$ and shifted up 3

$$y = 10^{-(x-4)}$$

$$y = \frac{1}{4}(10)^x + 3$$

6. Graph the following exponential functions. Mark all points accurately within the domain provided.

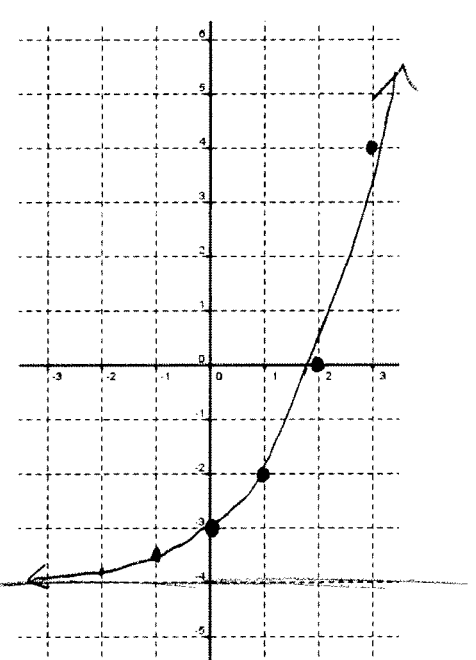
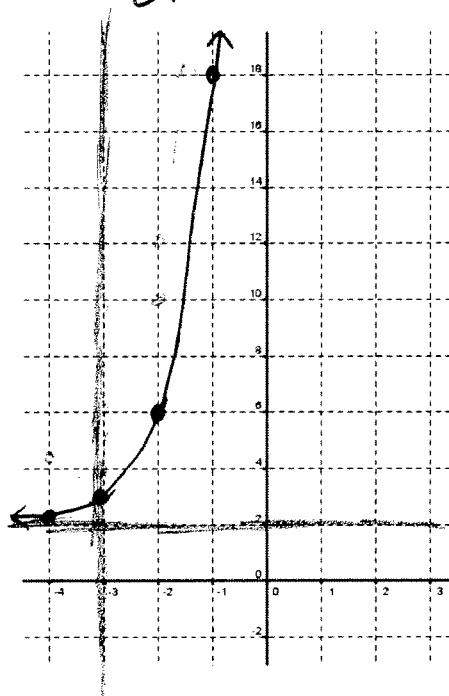
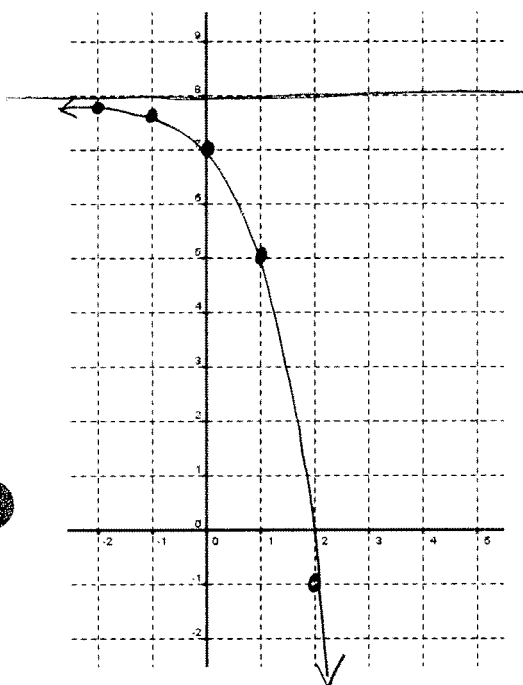
a) $y = 8 - 3^x$

$$= -3^x + 8$$

b) $y = 4^{x+3} + 2$

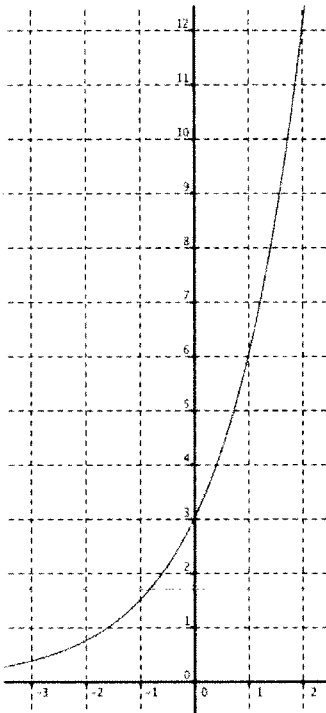
BE CAREFUL OF GRID!

c) $y = 2^x - 4$

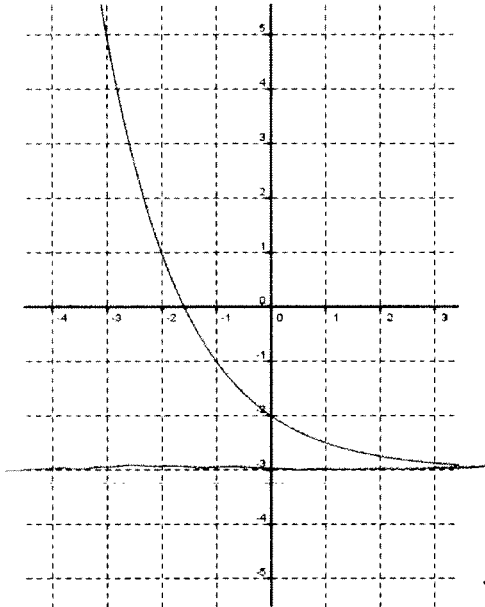


7. Write an equation for each of the following exponential functions. Answers may vary

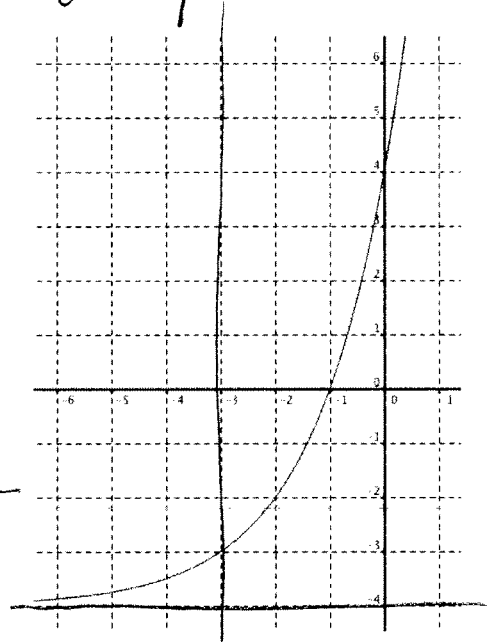
a) $y = 3(2)^x$



b) $y = 2^{-x} - 3$



c) $y = 2^{(x+3)} - 4$ ← same thing!
OR $y = 8(2)^x - 4$ ←



8. Earlier today, Gillian bought a Mercedes CLK, fully-loaded, for \$270 000. It is expected to depreciate in value by 20% per year. If she decides to sell it 10 years from now, how much will it be worth?

$$V = 270000(0.8)^{10}$$

$$= 28991$$

∴ In 10 years it will be worth \$28991

9. Ryan puts \$15 000 into a Tax-Free Savings Account (TFSA) that pays 6% per year (compounded monthly), and never touches it again. Assuming that he's 16 now, how much will the account be worth when he uses it at age 65?

$$\begin{aligned} A &= P(1+i)^n \\ &= 15000(1.005)^{588} \\ &= 281666.74 \end{aligned}$$

$$i = \frac{0.06}{12} = 0.005$$

$$n = 49 \times 12 = 588$$

\therefore By Age 65 it will be worth \$281666.74

10. Cerium-143 has a half-life of 33 hours. What mass of a 40 mg sample remains after 4 days?

$$\begin{aligned} A &= A_0 \left(\frac{1}{2}\right)^{t/h} \\ &= (40 \text{ mg}) \left(\frac{1}{2}\right)^{96/33} \\ &= 5.33 \text{ mg} \end{aligned}$$

$$4 \text{ days} \times \frac{24 \text{ hr}}{\text{day}} = 96 \text{ hrs}$$

\therefore 5.33 mg remains after 4 days.