MCR3U 1.5 Solving quadratic equations

We can solve any quadratic equation by determining its roots (the input values that make the function equal to zero):

$ax^2+bx+c=0$ roots: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ (quadratic formula)

There may be 0, 1, or 2 solutions to a quadratic equation, depending on the values of a, b, and c.

Ex. Determine the x-intercepts of the following quadratic functions. Then graph each function.

a) $f(x)=-x^2-8x-10$ b) $g(x)=4x^2-12x+9$ c) $h(x)=2x^2-24x+73$

a) $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{8\pm\sqrt{(-8)^2-4(-1)(-10)}}{2(-1)}$

$=\dfrac{8\pm\sqrt{24}}{-2}$

$=-6.45$ or -1.55

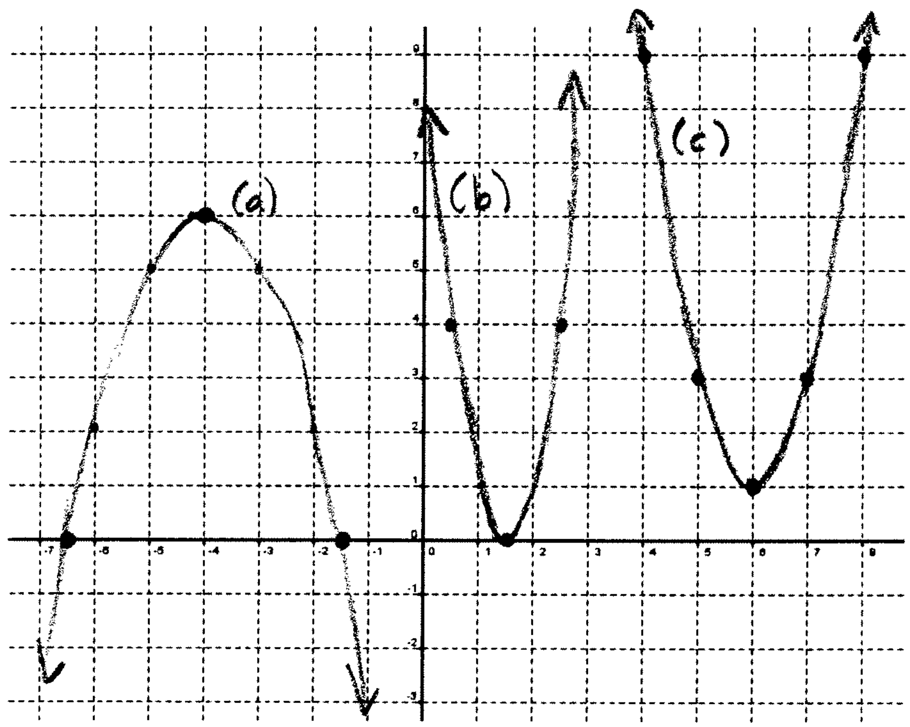
Vertex:

$x=\dfrac{-b}{2a}=\dfrac{8}{2(-1)}=-4$

$f(-4)=-(-4)^2-8(-4)-10$

$=6$

$\therefore V(-4,6)$



b) $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{12\pm\sqrt{144-4(9)(4)}}{2(4)}$

$=\dfrac{12\pm0}{8}$

$=1.5$ or 1.5

One x-intercept $\Rightarrow$ also the vertex!

Use step pattern to generate more points

c) $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{24\pm\sqrt{(-24)^2-4(2)(73)}}{2(2)}$

$=\dfrac{24\pm\sqrt{-8}}{4}$

NO ANSWER

Vertex: $x=\dfrac{-b}{2a}=\dfrac{24}{2(2)}=6$

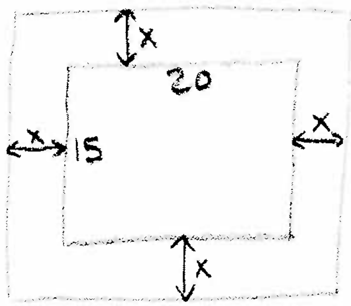
$h(x)=2(6)^2-24(6)+73=72-144+73=1$

$V(6,1)$

Use step pattern to generate more points

We can use the quadratic formula to solve problems involving any quadratic equation, as long as we rearrange the equation so that it is equal to zero.

Ex. A landscape design company is building a storage shed that is 15 ft by 20 ft in a client's backyard. The shed will be surrounded by interlocking brick forming a pathway of equal width on all sides of the shed. If the budget allows for the purchase of 400 sq ft of brick, determine how wide the pathway will be around the shed.



Brick path of uniform width (x)

$A_{shed} = 15 \times 20 = 300$

$A_{brick} = 400$ (given)

$A_{total} = (20+2x)(15+2x)$

$= 300 + 30x + 40x + 4x^2$

$= 4x^2 + 70x + 300$

$A_{shed} + A_{brick} = A_{total}$

$300 + 400 = 4x^2 + 70x + 300$

$0 = 4x^2 + 70x - 400$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-70 \pm \sqrt{(70)^2 - 4(4)(-400)}}{2(4)}$

$= \frac{-70 \pm \sqrt{11300}}{8}$

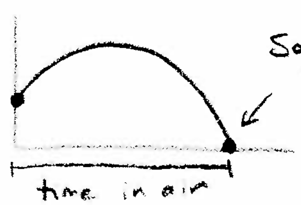
$= 4.54$ or -13.3

∴ The brick border will be 4.54 m wide

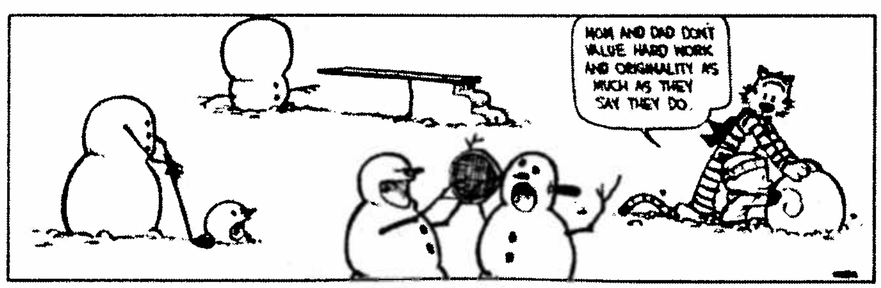
Ex. Calvin's "snowman sports scene" shows that diving into snow is not quite the same as diving into water. Calvin has modelled the height of the diving snowman's head by the quadratic function:

$h = -4.9t^2 + 3.92t + 2.216$ — h = height of diver's head above ground (metres)
t = time from start of dive (seconds)

How long was the snowman's head in the air during the dive (to the nearest tenth of a second).



Solve for x-intercept



$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3.92 \pm \sqrt{(-3.92)^2 - 4(-4.9)(2.216)}}{2(-4.9)}$

$= \frac{-3.92 \pm \sqrt{58.8}}{-9.8}$

$= 1.18$ or -0.38

∴ It was in the air for 1.18 s.

Homework: p. 49 # 3acf, 5acd, 12, 13b, 15, 16.