MCR3U 1.3 Maximum or minimum of a quadratic function

We can graph a quadratic function by determining its axis of symmetry, vertex, and step pattern if its equation is given in standard form:

$f(x) = ax^2 + bx + c$

axis of symmetry: $x = \dfrac{-b}{2a}$

vertex: evaluate $f(x)$ at the axis of symmetry

step pattern: 1,3,5,7,... multiplied by the value of a

Ex. Graph the following parabolas and label each axis of symmetry, vertex, maximum, and minimum.

a) $f(x) = x^2 + 14x + 50$

$x = \frac{-b}{2a} = \frac{-14}{2} = -7$

$f(-7) = (-7)^2 + 14(-7) + 50$
$= 49 - 98 + 50$
$= 1$

$V(-7, 1)$

b) $g(x) = 3x^2 + 6x - 3$

$x = \frac{-b}{2a} = \frac{-6}{2(3)} = -1$

$f(-1) = 3(-1)^2 + 6(-1) - 3$
$= 3 - 6 - 3$
$= -6$

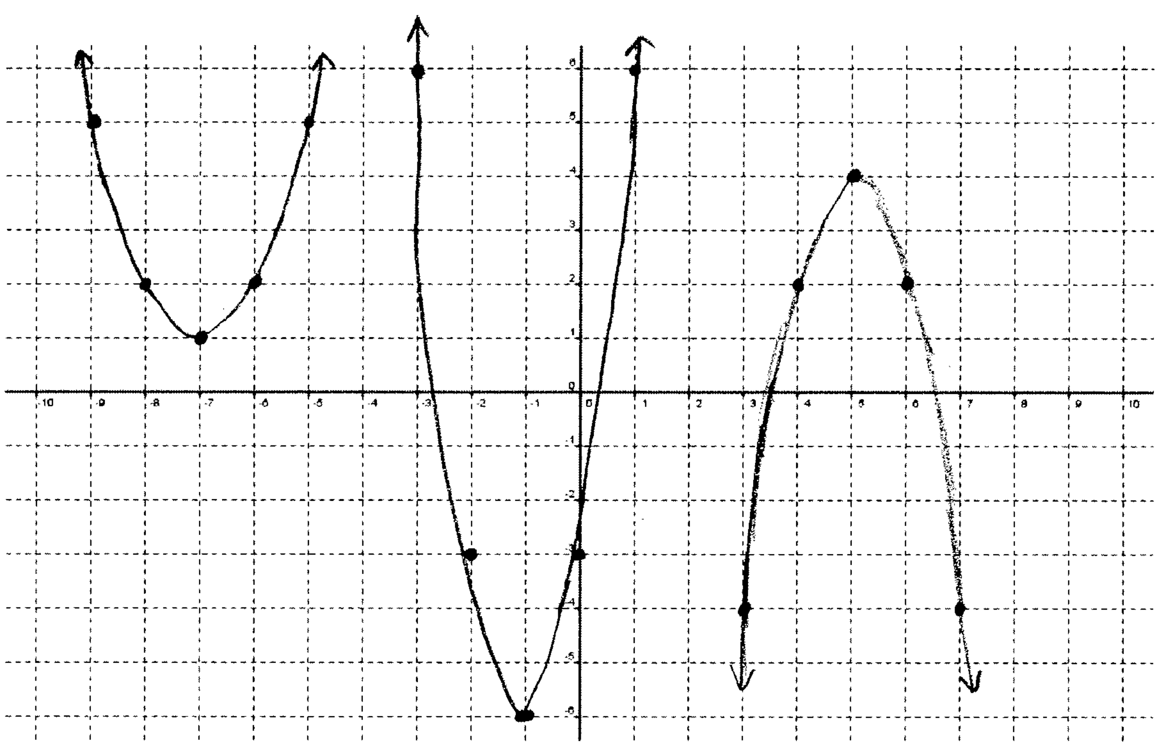
$V(-1, -6)$

c) $h(x) = -2x^2 + 20x - 46$

$x = \frac{-b}{2a} = \frac{-20}{2(-2)} = 5$

$f(5) = -2(5)^2 + 20(5) - 46$
$= -50 + 100 - 46$
$= 4$

$V(5, 4)$



We can use the axis of symmetry and vertex to solve problems involving maximum and minimum values of any quadratic function.

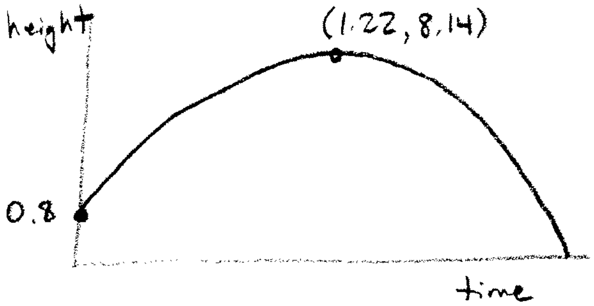
Ex. A soccer ball follows the path described by $h(t) = -4.9t^2 + 12t + 0.8$ where h represents the height (in metres) of the ball above the ground, and t represents the time in seconds after it was kicked.

a) Determine the maximum height the soccer ball reaches $\rightarrow$ Vertex
b) Determine the initial height from which the ball was kicked
c) Sketch a graph of the path followed by the ball

a) $x = \frac{-b}{2a} = \frac{-12}{2(-4.9)} = 1.22$ $\quad h(1.22) = -4.9(1.22)^2 + 12(1.22) + 0.8 \doteq 8.14\,m$

b) At $t=0$, $h(0) = -4.9(0)^2 + 12(0) + 0.8 = 0.8\ m$

c)



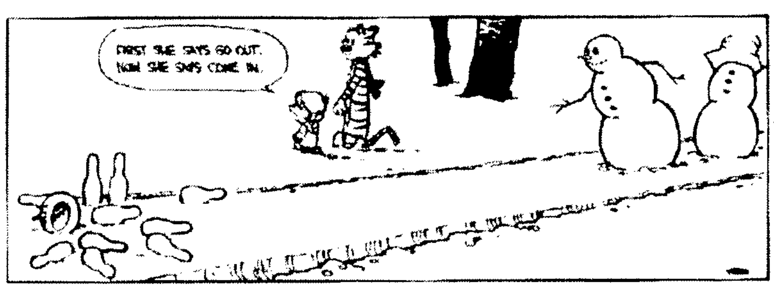
Ex. A trucking company has modeled the monthly fuel cost for each truck $C(v) = 0.0035v^2 - 0.63v + 160$ where v represents the driving speed (in km/h) on the highway.

a) Determine the most efficient speed for the company to operate their trucks. (lowest C)
b) Determine the minimum monthly fuel cost for each truck. $\hookrightarrow$ vertex

a) $x = \frac{-b}{2a} = \frac{0.63}{2(0.0035)} \doteq 90$ km/h

b) $C(90) = 0.0035(90)^2 - 0.63(90) + 160$
$= \$131.65$

Ex. Calvin and Hobbes run a bowling alley in the winter. The alley is usually booked for 200 hours each week at a rate of \$10 per hour. Calvin wants to raise the hourly rate to maximize his revenue. Market research shows that for every \$0.50 increase he will lose 4 hours of business each week.



a) Write an equation for his expected revenue after x number of \$0.50 increases. $(200-4x)(10+0.5x)$
$= 2000 - 40x + 100x - 2x^2$
$= -2x^2 + 60x + 2000$
b) Determine the hourly rate that will maximise revenue.
c) Determine the maximum revenue. $\quad x = \frac{-b}{2a} = \frac{-60}{2(-2)} = 15$

$\therefore$ 15 increases

Homework: p. 31 # 2ace, 3bdf, 6, 7, 8, 9. $\quad \therefore Cost = 10 + 0.5(15) = 17.50$

c) $R = -2(15)^2 + 60(15) + 2000 = \$2450$