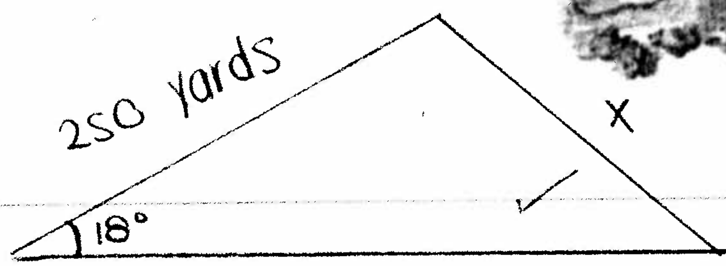
Cosine law: $c^2 = a^2 + b$

3 /3 A

1. The green on a golf hole lies 400 yards directly east of the tee, with a water hazard in be and the green. If a golfer's first shot travels 250 yards from the tee at an angle of 18° [t east] and lands on the fairway, how far will the golfer have to hit the second shot from tl order to land on the green? Include a diagram.



$400^2 + 250^2 - 2(400)(250)(\cos 18)$

$x^2 = 32{,}288.7$

$x = 179.7$ ✓

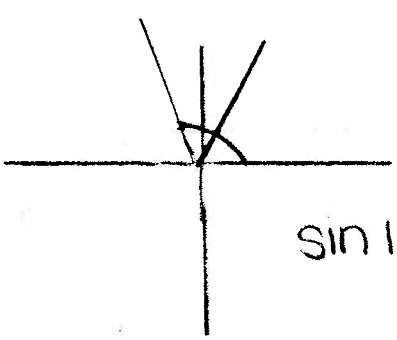
∴ the golfer will have to hit the secon
179.7 yards. ✓

2/2 T

2. Determine another angle between 0 and 360 degrees that has the same trigonometric ra angle given.

360−

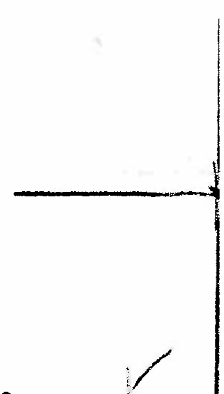
a) $\sin 110°$ $180-110=70$



✓

$\sin 110° = \sin 70°$

b) $\cos 40°$



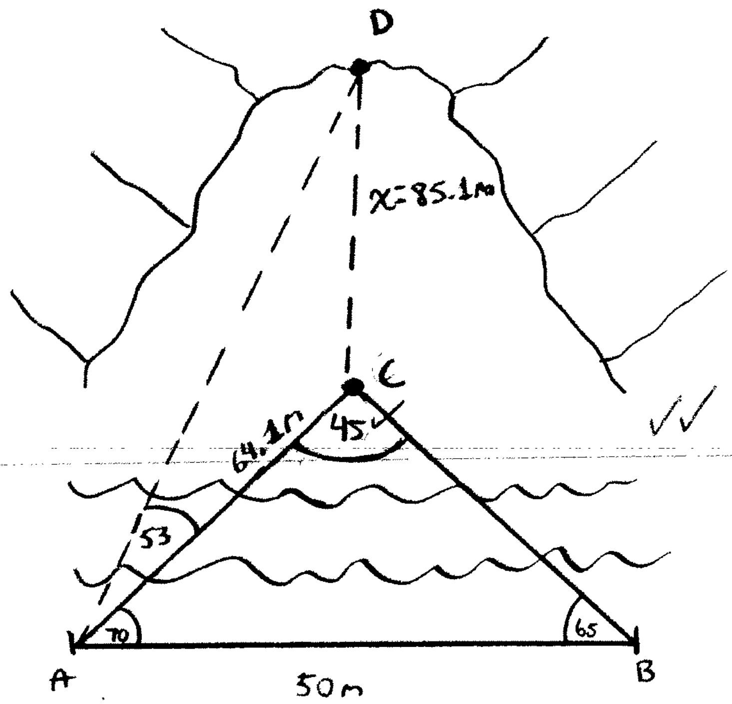
✓

$\cos 40° = \cos 3$

5 /5 A

2 /2 C

3. A surveyor is on one side of a river and wants to measure the height of a cliff on the other side of the river. She measures a baseline of 50 m from A to B and then measures angle ABC to be 65 degrees when point C is at the base of the cliff. She walks to point A and measures angle CAB to be 70 degrees. Then the angle of elevation (angle CAD) is 53 degrees to point D on the top of the cliff (above point C). Draw a diagram to represent this situation and determine the height of the cliff.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 65}{b} = \frac{\sin 45}{50} \checkmark$$

$$\sin 45\, b = \sin 65 (50)$$

$$b = \frac{\sin 65(50)}{\sin 45}$$

$$b = 64.1 \checkmark$$

$$\tan 53 = \frac{x}{64.1} \checkmark$$

$$64.1 \tan 53 = x$$

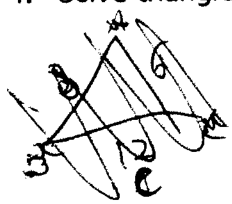
$$85.1 = x \checkmark$$

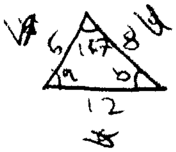
∴ The height of the cliff is 85.1 m.

3/3 K

2/2 T

4. Solve triangle ABC if: a = 8 km, b = 6 km, c = 12 km.





$c^2 = a^2 + b^2 - 2ab\cos C$

$12^2 = 6^2 + 8^2 - 2(6 \times 8)\cos C$ ✓

$144 = 100 - 96\cos C$

$44 = -96\cos C$

Nice!

$C \approx 117.3^\circ$ (obtuse) ✓

$\frac{\sin 117.3}{12} \quad \frac{\sin b}{6} = \frac{\sin a}{8}$ ✓T

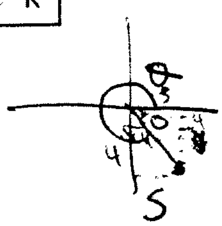
$b = 26.4^\circ$ ✓T OR $180 - 36.3 = 143.3$

$c = 36.3^\circ$ ✓

$26.4 + 36.3 + 117 = 180^\circ$

3/3 K

5. Determine an exact value for $\cos\theta$ if the point $(5, -4)$ lies on the terminal arm of θ.



$r = \sqrt{x^2 + y^2}$

$r = \sqrt{4^2 + 5^2}$

$= \sqrt{16 + 25}$ ✓

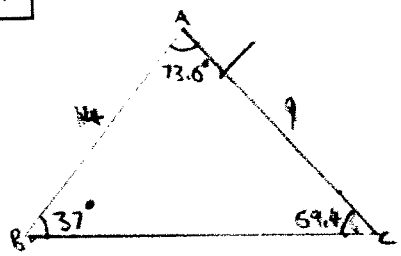
$= \sqrt{41}$

$\cos\theta = \frac{5}{\sqrt{41}}$ ✓ ✓

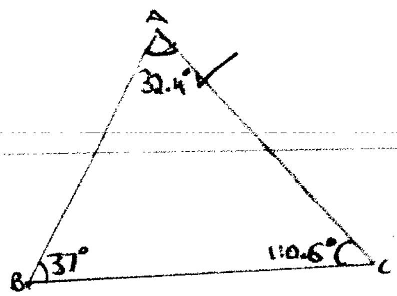
$3\ {}^{/3}_{K}$

$2\ {}^{/2}_{T}$

6. Determine the missing angles in triangle ABC if: B = 37°, b = 9 m, c = 14 m.



or



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 37^\circ}{9} = \frac{\sin C}{14}$$

$$\frac{14(\sin 37)}{9} = \frac{9(\sin C)}{9}$$

$$\sin C = 0.9362$$

$$C = 69.4^\circ$$

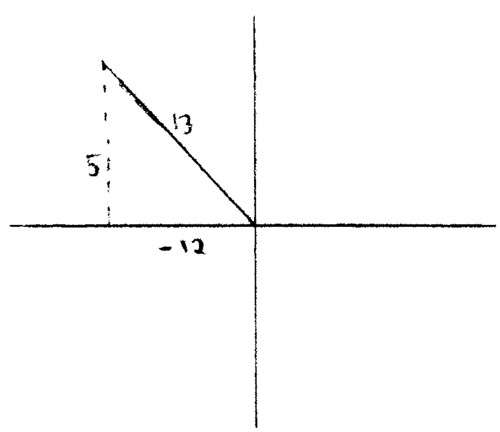
or

$$C = 110.6$$ YES!

Perfect!

$4\ {}^{/4}_{T}$

7. If the angle θ lies in the second quadrant and $\sin\theta = \frac{5}{13}$, determine an exact value for $\tan\theta$.



$$a^2 + b^2 = c^2$$
$$a^2 + 5^2 = 13^2$$
$$a^2 = 13^2 - 5^2$$
$$a^2 = 169 - 25$$
$$a^2 = 144$$
$$\sqrt{a^2} = \sqrt{144}$$
$$a = 12$$

$$\boxed{\tan\theta = \frac{5}{-12}}$$

Nice

$\boxed{2 \; /2 \; C}$

8. Explain why there are two angles between 0 and 360 degrees with the same sine ratio.

Sin is positif in quadrants 1 and 2 !

$\sin(45) = \frac{1}{\sqrt{2}}$ $\quad \sin(135) = \frac{1}{\sqrt{2}}$

$180 - 45 = 135$

$\boxed{5 \; /5 \; K}$

9. Determine the exact value of each trigonometric ratio.

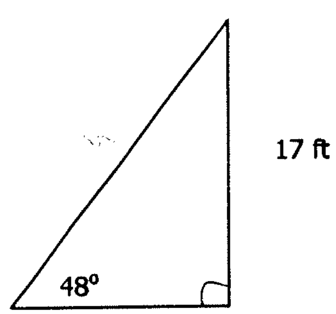
a) $\sin 45^{\circ} = \frac{1}{\sqrt{2}}$

b) $\cos 150^{\circ} = -\frac{\sqrt{3}}{2}$

c) $\tan 240^{\circ} = \sqrt{3}$

$\boxed{3 \; /3 \; K}$

10. Determine the unknown side in this right angle triangle using a reciprocal trigonometric ratio.



x = 15.3 ft

$\cot 48^{\circ} = \frac{x}{17}$

$\frac{1}{\tan} 48^{\circ} = \frac{x}{17}$

$\tan 48 = \frac{17}{x}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta = 1$	$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

7/4 K

1/2 T

11. Use the basic identities provided above to prove the following identities.

a) $\cot\theta \sin\theta \sec\theta = 1$

$LS = \cot\theta \sin\theta \sec\theta$

$= \frac{1}{\tan\theta}(\sin\theta)\left(\frac{1}{\cos\theta}\right)$ ✓

$= \frac{\cos\theta}{\sin\theta}(\sin\theta)\left(\frac{1}{\cos\theta}\right)$

$= \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}\left(\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}\right)$ 2/2

$= 1$ ✓

$RS = 1$

$\therefore LS = RS$

b) $\sin\theta(1+\tan\theta) = \tan\theta(\sin\theta + \cos\theta)$

$LS = \sin\theta\,(1 + \tan\theta)$

$= \sin\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right)$ ✓

$= \sin\theta\left(\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{\sin\theta\cos\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$ ✓

$RS = \tan\theta\,(\sin\theta + \cos\theta)$ ✓

$= \frac{\sin\theta}{\cos\theta}(\sin\theta + \cos\theta)$

$= \frac{\sin^2\theta}{\cos\theta} + \frac{\sin\theta\cos\theta}{\cos\theta}$ ✓

$\therefore LS = RS$